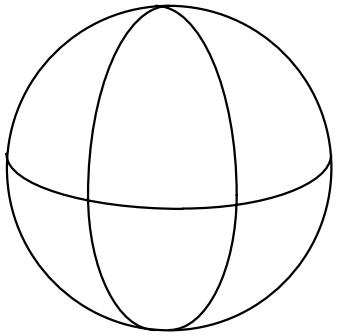
A Model of
Spherical (Riemannian) Geometry



Tools: globe, string, tracing paper, water-soluble pens, protractors,

The purpose of this experiment is to study the results if a plane were really the surface of a huge sphere, and not *flat* as we envision it.

Model:

The surface of a sphere will be our model for this geometry.

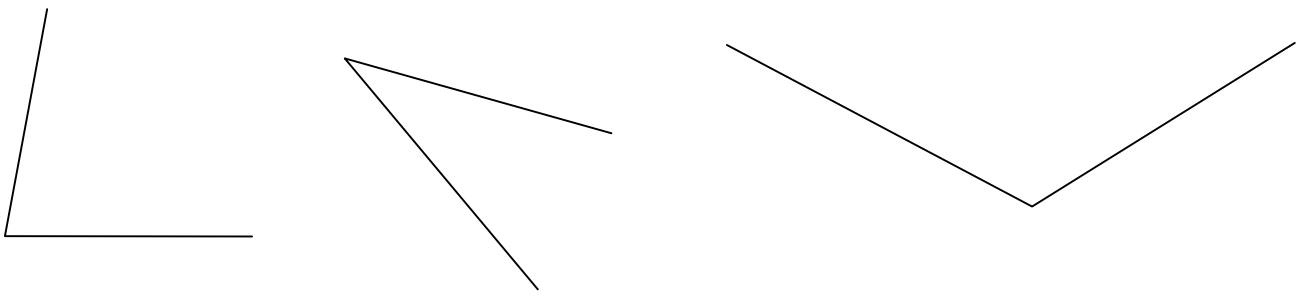
Definitions:

great circle: A great circle is a line on a sphere that divides the sphere into two equal halves. The equator is an example of a great circle.

straight line: The shortest distance between two points.

Parallel lines: Two lines are parallel if they never intersect

Practice measuring these angles with your protractor. Align the *dot* (at 0°) with the vertex of the angle, one side with the line on the protractor, and read where the other side of the angle crosses the protractor.



1. **The shortest distance between 2 points.**

a. Find the shortest distance between Austin, Texas, and Tehran, Iran.

- Find Austin, Texas, and Tehran, Iran.
- Use a taut string to help you find the shortest distance between these two points. Trace along this line with a water-soluble pen.
- Circle the location that is on the shortest path:

  Reykjavik, Iceland Denali, Alaska The north pole

- Now extend the string around the rest of the sphere. Trace the line. What appears to happen?

b. Find the shortest distance between Boston, Massachusetts and Beijing, China.

- Find Boston, Massachusetts and Beijing, China.
- Use a taut string to help you find the shortest distance between these two points. Trace along this line with a water-soluble pen.
- Circle the location that is on the shortest path:

  Reykjavik, Iceland Denali, Alaska The north pole?

- Now extend the string around the rest of the sphere. Trace the line. What appears to happen?

c. Find the shortest distance between Austin, Texas, and Beijing, China.

- Find Austin, Texas and Beijing, China.
- Use a string to help you find the shortest distance between these two points. Trace along this line with a water-soluble pen.
- Circle the location that is on the shortest path:

  Reykjavik, Iceland Denali, Alaska The north pole?

- Now extend the string around the rest of the sphere. Trace the line. What appears to happen?

d. What information can you generalize about *straight lines* on a sphere?

2. Parallel lines

- Draw a *straight line* (great circle) on your sphere.
- Draw another point on your sphere.
- Draw another *straight line* (great circle) through that point that is parallel to the original straight line.
- What observations can you make about parallel lines if a plane is really the surface of a sphere?

- Why aren't lines of latitude on a globe parallel lines?

3. The sum of the angles of a triangle.

On the globe, for each set of cities

- Draw triangles whose vertices are the cities.
- Trace each angle carefully on the paper provided.
- Extend the sides carefully using a straight edge and measure the angle.
- Record the angle measurements in the table below
- Try to draw the triangle as it appears in three-dimensional space.
- Erase the triangle

Cities	Angle *1*	Angle *2*	Angle*3*	Total	Draw the triangle
Minneapolis, Minnesota Austin, Texas Boston, Massachusetts					
Panama City, Panama Nome, Alaska, Dublin, Ireland					
Quito, Ecuador Monrovia, Liberia Thule, Greenland					
Quito, Ecuador Bangkok, Thailand The South Pole					
Wellington, New Zealand Moscow, Russia Rio de Janeiro, Brazil					

Mark the cities that gave you the smallest and the largest triangle.

What conclusion can you make about the sum of the angles of a triangle when that triangle lies on a sphere?

Can you observe the relationship between the size of the triangle and the sum of the angles of the triangle?