

Some of Fermat's Investigations

The next three statements are statements made by Fermat that were later proven. Explain what each statement means. Give examples of these statements.

1. A prime of the form $4n + 1$ can be expressed as the sum of 2 squares.
2. Every non-negative integer can be expressed as the sum of four or fewer squares.
3. Every non-negative integer can be expressed as the sum of four or fewer squares.

The next two statements are conjectures in number theory that have not been proven. Explain what each statement means. Give examples of each statement.

4. **The twin prime question.** Are there infinitely many pairs of prime numbers that differ from one another by two
5. **The Goldbach Question.** Can every positive, even number greater than 2 be written as the sum of two primes?