Origami: More Than Just Art!

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Presentation Outline

- How is origami a powerful mathematical tool?
- What can origami do in the classroom?
- How can we incorporate origami as a mathematical tool in our classrooms?
How Is Origami A Powerful Mathematical Tool?

- Origami can be used to construct geometric objects.
  - All 5 of the Platonic Solids
    - Tetrahedron
    - Hexahedron
    - Octahedron
    - Icosahedron
    - Dodecahedron

- Origami is an unconventional method for unraveling famous mathematical problems.
  - 2 of the 3 problems of antiquity
    - Trisection of an arbitrary angle
    - Doubling of a cube
The Platonic Solids

- What is a platonic solid?
  - “The Platonic solids, also called the regular solids or regular polyhedra, are convex polyhedra with equivalent faces composed of congruent convex regular polygons” (source 3).
- Each of the 5 platonic solids can be created using origami.
The Problems Of Antiquity

What are the problems of antiquity?

“Three geometric questions raised by the early Greek mathematicians [that] attained the status of classical problems in Mathematics. These are:

- Doubling of the cube
  - Construct a cube whose volume is double that of a given one [using only a straightedge and compass].

- Angle trisection
  - Trisect an arbitrary angle [using only a straightedge and compass].

- Squaring a circle
  - Construct a square whose area equals that of a given circle [using only a straightedge and compass]” (source 4).
What Can Origami Do In The Classroom?

- Origami...
  - “strengthens geometry and spatial relation skills.”*
  - “promotes the study of open-ended problems.”*
  - encourages individual comprehension “of concepts and [the] integration of new ideas with old perceptions.”*
  - creates opportunity for “peer teaching and learning.”*
  - increases “mathematical discourse.”*
  - AND MUCH MORE...

*from source 5
How Can We Incorporate Origami As A Mathematical Tool In Our Classrooms?

Geometric figures of origami serve as great manipulatives for studying solid geometry.

- “We live in a three-dimensional world, yet most of a learner’s mathematical education is in two dimensions.”*

- We “see worksheets handed out with images of three-dimensional shapes rather than having a cube or a cylinder for students to pick up and hold in space, ...[they are asked] to find the volume, the total surface area, and the lateral surface area by looking at a two-dimensional drawing on a sheet of paper.”*

*Source 6
Symmetry Of The Platonic Solids

“A symmetry of a polyhedron is a way of moving the polyhedron so that it occupies the same physical space as before it was moved”

“As an example, consider the symmetries of the cube.”

“A cube can be rotated 90°, 180°, or 270° around an axis passing through the centers of two opposite faces of the cube”:

“A cube can be rotated 120° or 240° around an axis passing through two opposite vertices of the cube”:

*All Source 7
Symmetry Of The Platonic Solids

Cont.

- “A cube can be rotated 180° around an axis passing through the midpoints of two opposite edges of the cube”:

- This can be done in the classroom by having student use the origami Platonic Solids and straws to represent the axis of symmetry.

*All Source 7*
Dualism With The Platonic Solids

- Dualism is a property with Platonic Solids where “the centers of the faces of any regular polyhedron form the vertices of another regular polyhedron.”

- “If we take the centers of the faces of a cube and connect them with edges and faces, we obtain an octahedron.”

- “The reverse is also true; if we take the centers of the faces of an octahedron and connect them with edges and faces.”

*Source 7*
Euler’s Formula Using Origami

- Students can discover the relationship between the vertices, edges, and faces of the Platonic Solids.
- This will lead them to the development of Euler’s Formula: \( V - E + F = 2 \).
Trisecting an arbitrary angle

Is it possible to trisect an arbitrary angle using only a straight edge and compass?

- No, it is impossible to do this using a straight edge and compass

H. Abe developed a method to trisect an angle using origami.
Abe’s Trisection:

(1) Let the angle you want to trisect originate from the lower left corner. Call this angle A. Make two parallel, equidistant horizontal creases at the bottom.

(2) Fold p1 onto L1 and p2 onto L2. This isn’t easy to do!
Abe’s Trisection Continued:

(3) With this folded, refold crease \( L_1 \), now in its new position, and extend it all the way up. This new crease, \( L_3 \), is the crease we want. Unfold step 2 and extend crease \( L_3 \) to the lower left corner (it should hit it!). The crease \( L_3 \) will mark the angle \( \frac{2A}{3} \).

Trisection pictures and steps from source 2.
Can We Prove Abe’s Trisection Works?

(1) Show that $AB = BC = CD$

(2) Then show that triangle $AOB$ is congruent to triangle $BOC$ which is congruent to triangle $COD$.

(3) This proves that $\angle AOB = \angle BOC = \angle COD$.

*All from Source 5
Doubling the cube

Is it possible, using only a straightedge and compass, to double the volume of a cube?

No, it is impossible to do this using a straight edge and compass

Messer developed a method to double a cube using origami.
Messer’s Doubling:

(1) Fold a square into thirds. Let p1, p2 and L1, L2 be as shown.

(2) Perform axiom (O6). Denote lengths X and Y where the point p1 lands on line L1. Then the ratio X/Y is the desired number, the cube root of two.

Doubling pictures and steps from source 2.
Messer’s Doubling Continued:

- Why does this work?
  - Well, the reason that it is impossible to double a cube with a straightedge and compass is because it is impossible to construct a line of length $\sqrt[3]{2}$.
  - Using Messer’s method, we can construct a line of length $\sqrt[3]{2}$.
Presentation Sources

1. http://members.tripod.com/~PeterBudai/Origami/Pictures_en.htm
5. COET95 Second International Conference on Origami in Education and Therapy
7. http://www.math.ohio-state.edu/~goldstin/origami/displaytext.html#inscribed