Backward Design Overview

What should students know, understand, and be able to do by the end of this lesson?

The student will be able to form a conjecture based on the performance of an experiment. The student will be able to find and explain patterns in Pascal's Triangle, including calculating the next row in Pascal's Triangle. The student will also be able to calculate combinatorics using Pascal's Triangle. The student will be able to expand a binomial expression to any power using Pascal's Triangle.

The students will show mastery of the content through working in groups and explaining what their conjectures are from the experiment.

What will you accept as evidence?

After the students have worked individually on an experiment and they have worked on a different experiment in each of the groups of 4, they will be able to explain in a journal that Pascal's Triangle can be used to perform calculations, such as combinatorics or expanding a binomial expression to any power. Also, the students will explain that the numbers in Pascal's Triangle correspond to the coefficients in the expansion of a binomial expression.

Essential Question(s):

Given a binomial expression, explain how you would expand the expression if the expression is raised to a large power, such 50? What's an easier way of performing the expansion using technology? Would Maple help?

Standards

New York State Learning Standards and Key Ideas
Mathematics, Science, and Technology- Standard 3

Students will:

- understand the concepts of and become proficient with the skills of mathematics;
- communicate and reason mathematically;
- become problem solvers by using appropriate tools and strategies;

through the integrated study of number sense and operations, algebra, geometry, measurement, and statistics and probability.

Problem Solving Strand:

*Students will build new mathematical knowledge through problem solving.*

A2.PS.3

Observe and explain patterns to formulate generalizations and conjectures

Algebra Strand

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

A2.A.36

Apply the binomial theorem to expand a binomial and determine a specific term for a binomial expansion.

Statistics and Probability Strand

*Students will understand and apply concepts of probability.*

A2.S.11

Calculate the number of possible combinations \( \binom{n}{r} \) of \( n \) items taken \( r \) at a time.

Objectives

Student-Centered Instructional Objectives and Bloom's Taxonomy
• Discover patterns in Pascal's Triangle, such as symmetry, finding the next row, each row is equal to one-half the next row, etc (Application).
• Express the coefficients of a binomial expansion as a combination (Synthesis).
• Expand a binomial expression using Pascal's Triangle (Application).

Materials
The materials that the teacher will need are as follow:

• A TI-83 calculator, or an approved calculator
• A copy of Pascal's Triangle on white paper
• A copy of Pascal's Triangle on an overhead
• A computer with Maple 10 installed
• White board markers (4 colors: Red, Black, Green, Blue)
• Pascal's Triangle and the Binomial Expansion homework sheet
• A task sheet, Expanding Binomials, to record the expansions of $(a + b)^n$, $n = 0, 1, 2, ...$

The materials that the teacher will need are as follow:

• A TI-83 calculator, or an approved calculator per student
• A copy of Pascal's Triangle (each student gets a copy)
• Pascal's Triangle and the Binomial Expansion homework sheet
• A task sheet, Expanding Binomials, to record the expansions of $(a + b)^n$, $n = 0, 1, 2, ...$

Student Adaptations

Adaptations

One of the students in the class has traumatic brain injury. Therefore, when giving instructions of what the class should do, I will direct all instructions to the class, so that the student is not left out and doesn't feel bad. Also, all instructions written on worksheets will be read out loud.

Lesson

The Anticipatory Set

"Before we start with today's lesson, I'd like to ask you a quick question to think about as we begin. If I were to say, expand $(a + b)^{100}$, I think you would kill me. If I tell you to expand $(a + b)^2$, you would hate me very much, then kill me. But if I tell you $(a + b)^2$, you would have mercy on me, because we should know how to expand this."
The Flow of Teaching and Learning Activities

Experimenting
As the students enter the room, I will direct their attention to the overhead, where they will see that they have been put into 6 groups that have already been assigned, and ask them to get into their groups; each group consists of 4 members. Each group will have already Pascal's Triangle (one per student) on the desk, and make sure that they know what the term "coefficient" means before starting.

After the students are in their groups, I will ask explain the directions carefully:

- Group 1 is to expand \((a + b)^0\), Group 2 expands \((a + b)^1\), Group 3 expands \((a + b)^2\), Group 4 expands \((a + b)^3\) Group 5 expands \((a + b)^4\), and Group 6 expands \((a + b)^5\).
- After each group has expanded the respective expression, I will ask a representative from each group to show the class (as a check) how to expand their respective expression on Maple 10. I will ask the representative from Group 6 to expand also \((a + b)^{100}\), in order for the students to see that it's not fun expanding \((a + b)^{100}\). I will assemble these expansions then on a worksheet and give it to them the next class. I will also ask the students to record \((a + b)^{100}\) in the their notes and to note that it is not fun or easy to expand this binomial.
- I will ask each group to look at Pascal's Triangle and try to find any patterns in the triangle, and maybe how some of them are related to the expressions expanded; I should emphasize that there are at least 7 patterns, so they shouldn't give up quickly.

Reflecting and Explaining
After each group has found as many patterns as possible in Pascal's Triangle, I will call on a member from each group to share one pattern that he or she found with the group. After each group has shared something, we will have a class discussion about how to find row 8 of Pascal's Triangle, since the handout given to them is up to row 7. Moreover, I'll tell the students to perform some combinations on their calculators, such as the ones that appear in Pascal's Triangle.

Hypothesizing and Articulating
We will have a class discussion on how the combinations relate to the coefficients of any expanded binomial expression, meaning that since each combination, such as \(\binom{4}{2}\) (Which is part of the coefficients of \((a + b)^4\)), is a number that is part of Pascal's Triangle, and since the coefficients in each expansion also correspond to a number to Pascal's Triangle, then each combination is one of the coefficients. Also, I will discuss with the class on how we can expand the variable \(a\) and \(b\). In \((a + b)^4\), we start with \(a^4\), which then decreases by a degree until it's \(a^0\). On the other hand, \(b\) will increase by a degree as a decreases.
Verifying and Refining
As a class, we will expand one example: \((a + b)^3\). The answer should be \(\binom{3}{0}(a^3)(b^0) + \binom{3}{1}(a^2)(b^1) + \binom{3}{2}(a^1)(b^2) + \binom{3}{3}(a^0)(b^3)\). We will then expand \((a + 2b)^3\) using combinations. To finish the lesson, I would make sure to emphasize to the class that this expansion will only work with \((a + b)^n\). To generalize, we will also expand \((a + b)^n\) using combinations.

Questions:
After the students have examined the patterns in Pascal's Triangle and they saw a connection between the numbers in Pascal's Triangle and combinations, I will ask the following question: How is \(\binom{4}{2}\) related to 6 in Pascal's Triangle? (Hint: Think about the row number and the entry).

Conclusion
On the board, I will write the following expressions: \((2a + b)^6\) and \((2a - b)^6\).

"As a Ticket to Leave (TTL), expand the two expressions above and write a sentence about their similarities and differences".

Homework Assignment:
Worksheet that consists of two parts. Part I is finding rows 8-12 of Pascal's Triangle, and Part II is more practice of expanding binomial expressions.
Pascal's Triangle

<table>
<thead>
<tr>
<th>row 0</th>
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<tbody>
<tr>
<td>row 1</td>
<td>1 1</td>
</tr>
<tr>
<td>row 2</td>
<td>1 2 1</td>
</tr>
<tr>
<td>row 3</td>
<td>1 3 3 1</td>
</tr>
<tr>
<td>row 4</td>
<td>1 4 6 4 1</td>
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<tr>
<td>row 5</td>
<td>1 5 10 10 5 1</td>
</tr>
<tr>
<td>row 6</td>
<td>1 6 15 20 15 6 1</td>
</tr>
<tr>
<td>row 7</td>
<td>1 7 21 35 35 21 7 1</td>
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</tbody>
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### Pascal's Triangle Using Binomial Expansion

<table>
<thead>
<tr>
<th>Row</th>
<th>Expression</th>
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<tbody>
<tr>
<td>0</td>
<td>$1$</td>
</tr>
<tr>
<td>1</td>
<td>$a + b$</td>
</tr>
<tr>
<td>2</td>
<td>$a^2 + 2ab + b^2$</td>
</tr>
<tr>
<td>3</td>
<td>$a^3 + 3a^2b + 3ab^2 + b^3$</td>
</tr>
<tr>
<td>4</td>
<td>$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$</td>
</tr>
<tr>
<td>5</td>
<td>$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$</td>
</tr>
<tr>
<td>6</td>
<td>$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$</td>
</tr>
<tr>
<td>7</td>
<td>$a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$</td>
</tr>
</tbody>
</table>
Pascal’s Triangle

<table>
<thead>
<tr>
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<th>( \binom{n}{2} )</th>
<th>( \binom{n}{3} )</th>
<th>( \binom{n}{4} )</th>
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</thead>
<tbody>
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<td>( \binom{5}{0} )</td>
<td>( \binom{6}{0} )</td>
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<td>( \binom{13}{0} )</td>
<td>( \binom{14}{0} )</td>
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</tbody>
</table>
Pascal’s Triangle and the Binomial Expansion

**Directions:**

**Par I:** Find rows 8-12 of Pascal’s Triangle

**Part II:** Using the Binomial Expansion and Pascal’s Triangle, expand the following:

1. \((a + b)^7\)
2. \((x + 3)^6\)
3. \((2x - y)^4\)
4. \((3x + 2y)^3\)